Modern Physics
-waves and particles-

Lecture notes by
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Wave-particle dualism

•At the end of the 19^{th} century there was a clear division among physical phenomena involving "waves" (electromagnetic, elastic …) and those involving particles. It was widely accepted that matter was composed by particles, physically identified as massive, although minuscule, basic entities (atoms, nuclei and electrons) while exchange and transport of energy could be realized through waves (like electromagnetic radiation).

•The discovery that the electromagnetic field can be seen also as an ensemble of particles having well-defined energy and momentum changed completely our understanding of the microscopic world, introducing the idea of the wave-particle dualism.

•The famous double-slit experiment demonstrating in 1801 the wave nature of light was repeated in 1909 by G. I. Taylor with a source of very weak intensity, generating on the average only a few photons per minute, in an irregular way: the interference pattern is realized (after a few months on the first experiment). So the wave nature is retained also at the single-photon level!

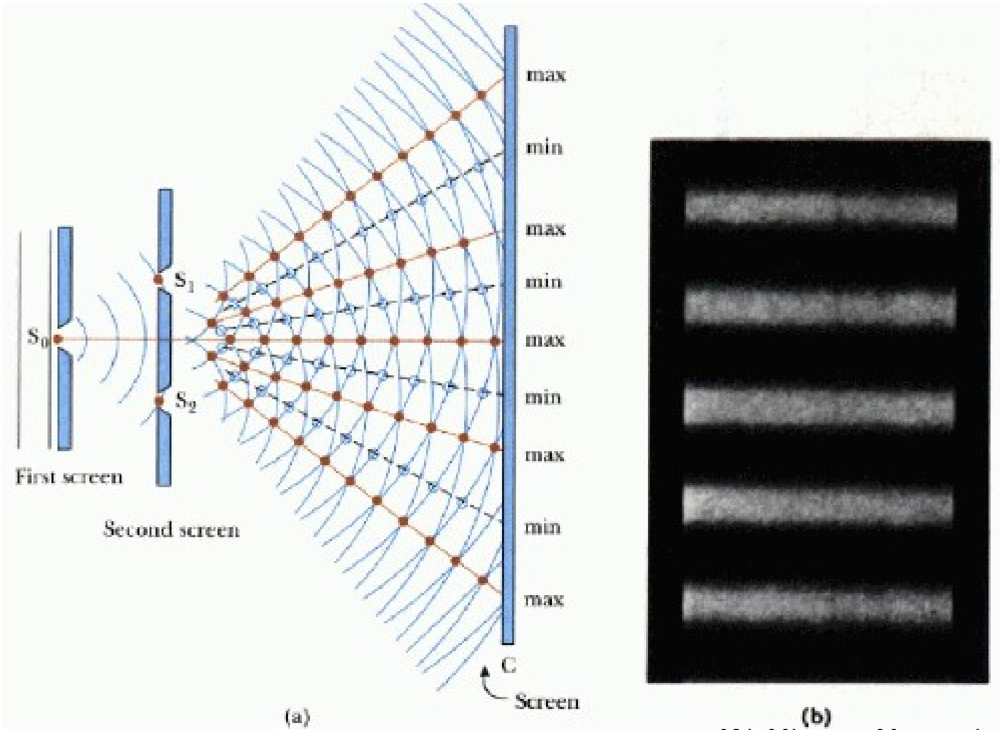
•The experiments of Ramsauer (electron-gas scattering cross-section, 1921) and Davisson and Germer (diffraction of electrons from crystals 1919) showed that also electrons could behave like waves, in spite of their nature of being "particles". A real 2-slit experiment with electrons could be performed only in 1961 (and with single electrons in 1989).

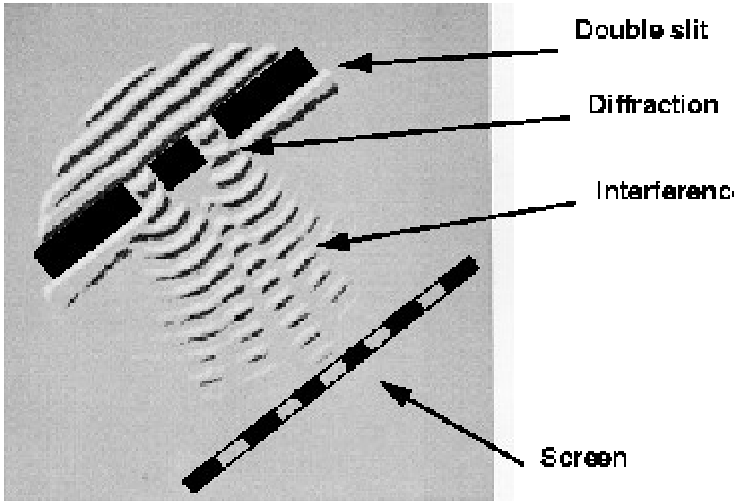
video on internet about 2-slit experiments (link)

Interference in photons

·The 2-slit set-up shows that light is subject to diffraction and interference, phenomena that can be understood assuming that light propagates as a wave of well-defined speed and wavelength.

·The fact that the interference pattern is reconstructed also when the light source is decomposed into single quantum of lights photons calls for a deep reconsideration of this phenomenon.





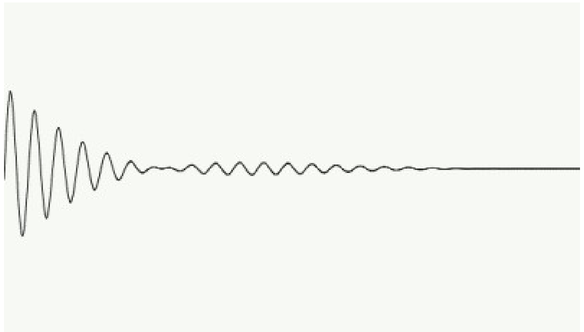
Double-slit diffraction and interference pattern

Probability waves

A sensitive detector would reveal the arrival of each photon on the screen positioned after the 2-slits in form of counting each energy transfer from one photon to the detector.
This would give us an irregular series of events, whose frequency over long acquisition times corresponds to the intensity of the electromagnetic field on the specific point. We can not know when the photon will come, but we can estimate the probability of counting a certain number of photons in a specified elapse of time.
We then arrive to a probabilistic description of an electromagnetic wave: we can associate a probability wave to each photon, which describes the probability to observe one photon of that energy in that volume of space in the unit time.

Our current view of the interference of the photon with itself can be summarized in this way:

1) Photons propagates like (probability) waves and the probability of observing one photon is proportional to the intensity of the electromagnetic field in each point of space;
2) We are only able to detect photons by an energy exchange process, which happens like a photon-matter particle interaction. The particle nature can be observed directly in the generation and absorption of photons.



A wave packet can be interpreted to be a "probability wave" describing the probability that a particle or particles in a particular state will be measured to have a given position and momentum. It is in this way similar to the wave function.

Wave character of the electron

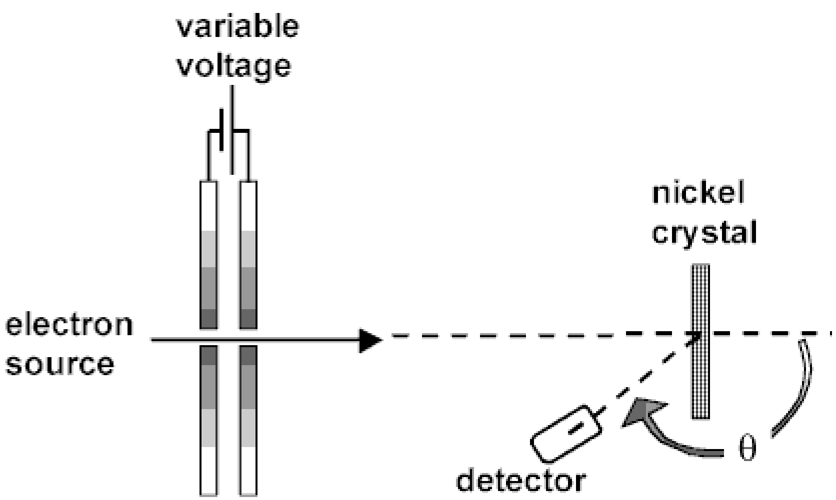
In 1919-1927 Davisson and Germer observed for the first time interference effects in reflected electrons from a crystal, uniquely determined by electron velocity, angle of observation and crystal orientation. This suggested that electrons can behave like waves.

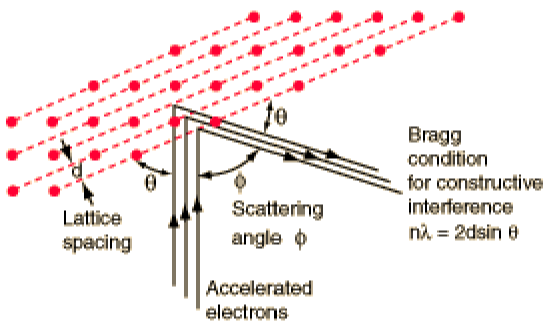
The suggestion by De Broglie (1924) was that duality must hold also for particles, which behaves like waves of well defined wavelength and momentum: $p = h/\lambda$.

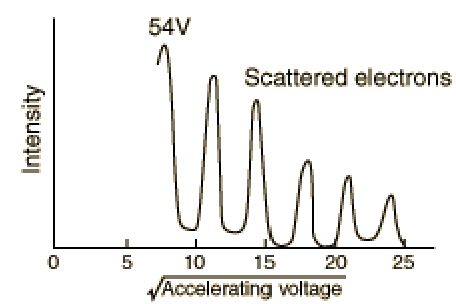
The relationship for electron between Energy and wavelength ($E = p^2/2m$) is then:

Electrons E (eV) = $151.3 / \lambda^2$ (Ang.)

Protons/neutrons E (meV) = $81.8 / \lambda^2$ (Ang.)



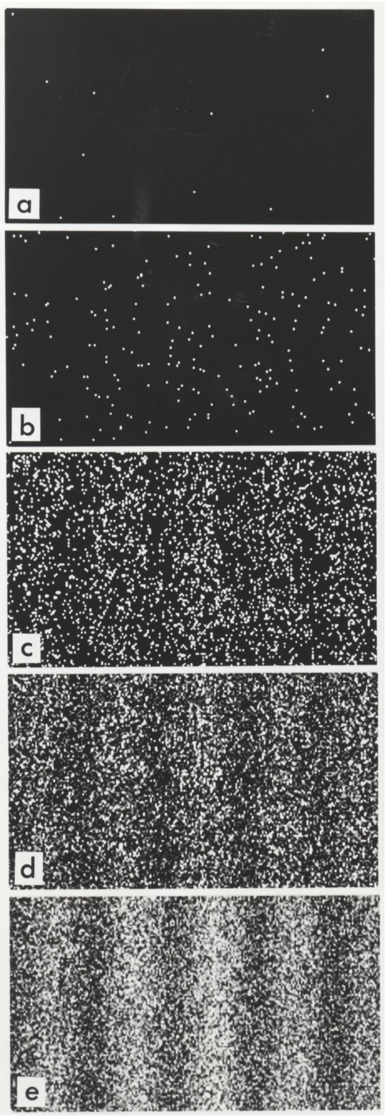




Single-particle two-slits experiments

Real single-particle experiments showing interference, similar to those performed with light were performed later.
A Young double slit experiment was not performed with anything other than light until 1961, when Clauss Jönsson of the University of Tübingen performed it with electrons, and not until 1974 in the form of "one electron at a time", in a laboratory at the University of Milan, by researchers led by Pier Giorgio Merli, of LAMEL-CNR Bologna.

The results of the 1974 experiment were published and even made into a short film, but did not receive wide attention. The experiment was repeated in 1989 by Tonomura et al. at Hitachi in Japan. Their equipment was better, reflecting 15 years of advances in electronics and a dedicated development effort by the Hitachi team. Their methodology was more precise and elegant, and their results agreed with the results of Merli's team. Although Tonomura asserted that the Italian experiment had not detected electrons one at a time—a key to demonstrating the wave-particle paradox—single electron detection is clearly visible in the photos and film taken by Merli and his group.



Wave packets

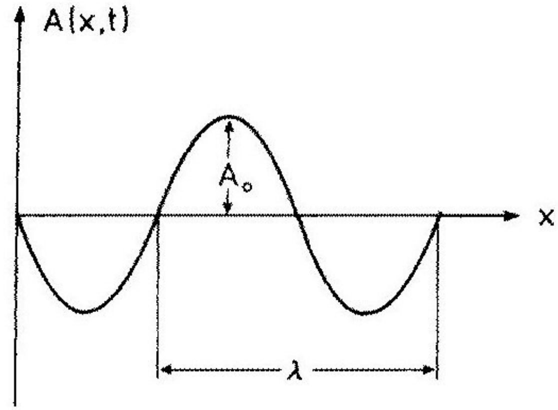


Fig. 7.1. Instantaneous view of a wave with amplitude A_0 and wavelength λ

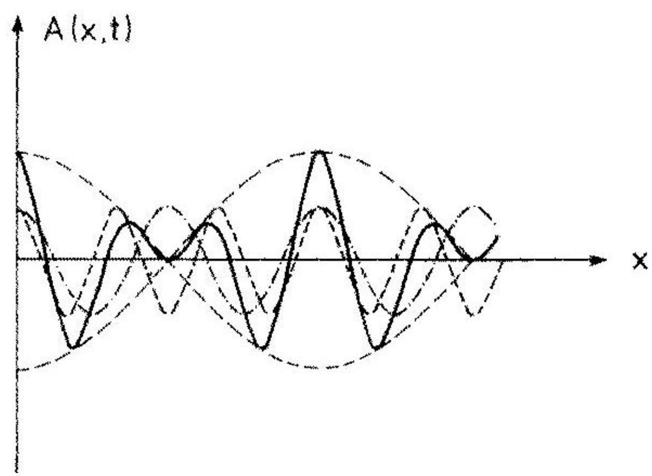


Fig. 7.2. Superposition of two waves of the same amplitude. Fundamental wave 1: (–·–), fundamental wave 2: (– –) same amplitude as 1. Resulting wave A: (——). The envelope $\cos(\Delta kx - \Delta\omega t)$ for constant t is also shown as a dashed curve

Presentation and calculations at the blackboard.

Wave packets and localization

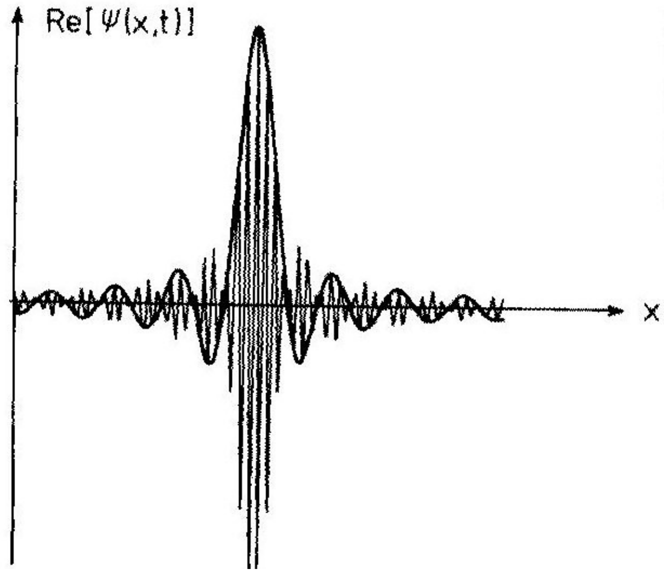


Fig. 7.3. The real part of $\psi(x,t)$ as a function of the position x. The rapid oscillations are described by $\cos(k_0 x - \omega_0 t)$ with t fixed. The envelope is given by $\sin[(\omega' t - x)\Delta k]/(\omega' t - x)$ with t fixed. Note that the scale of the x axis has been greatly reduced in comparison to Fig. 7.2

$$v_{group} = \frac{d\omega}{dk}$$

$$v_{phase} = \frac{\omega}{k}$$

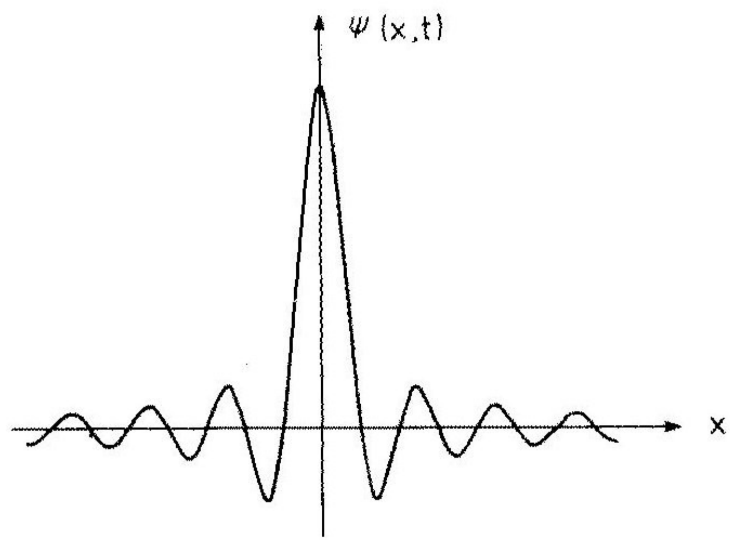


Fig. 7.4. The envelope of the real part of the wave packet (7.12) (Fig. 7.3). The first node is at $x_0 = \pi/\Delta k$

Presentation and calculations at the blackboard.

Probability waves

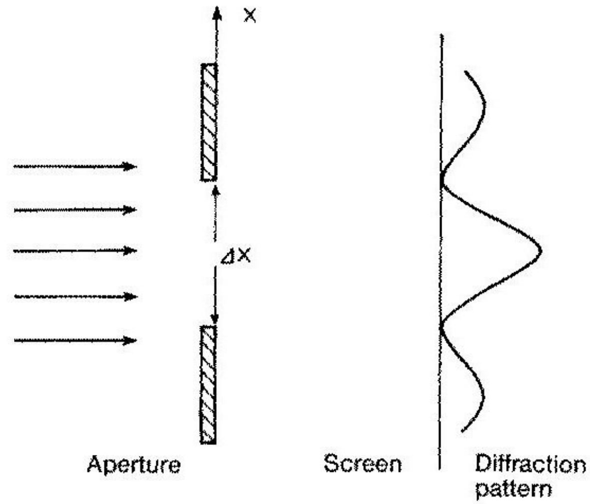


Fig. 7.5. An electron beam (arrows at left) passes through an aperture and generates a diffraction pattern on a screen. The intensity distribution on the screen is shown schematically on the right

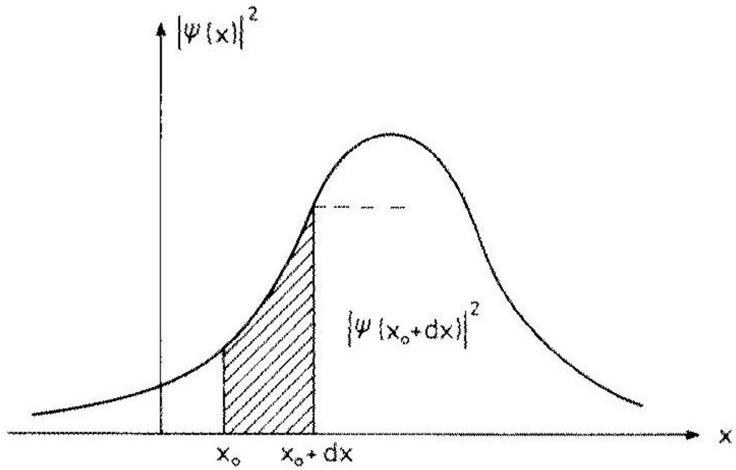


Fig. 7.6. $|\psi(x)|^2$ as a function of x at a given time t. The shaded area corresponds to the probability that the electron is located in the interval x_0 to $x_0 + dx$

Presentation and calculations at the blackboard.

Uncertainty principle

$$\frac{\Delta x}{2} \geq \frac{\pi}{\Delta k}$$

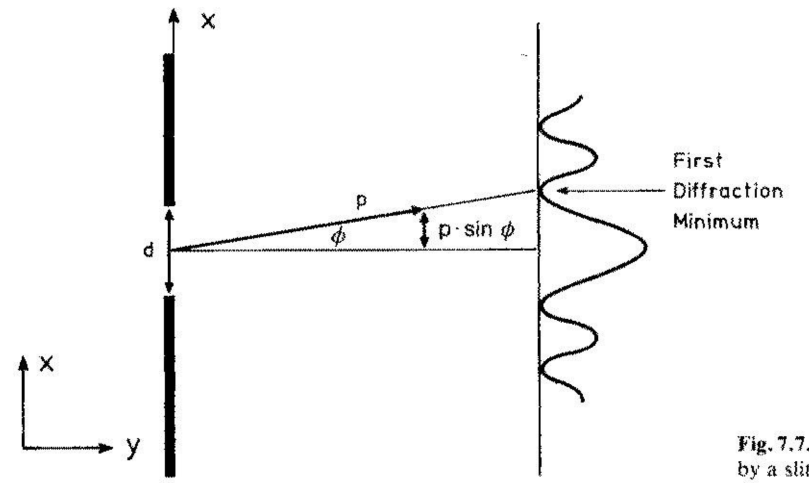


Fig. 7.7. Diffraction of a wave by a slit

$$\Delta x \Delta p \geq h$$

$$\Delta \omega \Delta t \geq 2\pi \Rightarrow \Delta E \Delta t \geq h$$

Presentation and calculations at the blackboard.